USN | | | | | | | | | | **06MAT31**

Third Semester B.E. Degree Examination, December 2010
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Find the Fourier series for the function $f(x) = x(2\pi - x)$ over the interval $(0, 2\pi)$ and deduce that $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. **(07 Marks)**

b. Obtain the half-range sine series for
$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{for } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{for } \frac{1}{2} < x < 1 \end{cases}.$$ **(07 Marks)**

c. Obtain the constant term and the co-efficients of sin θ and sin 2 θ in the Fourier expansion of y given the following data **(06 Marks)**

θ°	0	60	120	180	240	300	360
y	0	9.2	14.4	17.8	17.3	11.7	0

2 a. Obtain the finite Fourier sine transform of the function $f(x) = \cos kx$, where k is a non integer, over $(0, \pi)$. **(07 Marks)**

b. Find the Fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$. **(07 Marks)**

c. Find the inverse Fourier transform of e^{-u^2}. **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary functions from $Z = f(x + I t) + g(x - i t)$, where $i = \sqrt{-1}$. **(07 Marks)**

b. Solve by the method of separation of variables $p y^3 + q x^3 = 0$. **(07 Marks)**

c. Solve $(mz - ny) p + (nx - lz) q = ly - mx$. **(06 Marks)**

4 a. Derive the one – dimensional heat equation. **(07 Marks)**

b. Obtain the D'Almbert's solution of the wave equation $u_{tt} = c^2 u_{xx}$, subject to the condition $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(07 Marks)**

c. Solve the wave equation $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < \pi$, given $u(0, t) = u(\pi, t) = 0$; $u(x, 0) = 0$; $\frac{\partial u}{\partial t}(x, 0) = A(\sin x - \sin 2x), A \neq 0$. **(06 Marks)**

PART – B

5 a. Find the smallest and the largest roots of $e^x - 4x = 0$, correct to 4 decimal places by Newton – Raphson method. **(07 Marks)**

b. Solve by Gauss elimination method
$2x_1 + x_2 + 4x_3 = 12$; $4x_1 + 11x_2 - x_3 = 33$; $8x_1 - 3x_2 + 2x_3 = 20$. **(07 Marks)**

c. Find the largest eigenvalue and the corresponding eigenvector of the matrix by using power method :
$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ taking $[1, 1, 1]^T$ as the initial eigenvector, perform 5 iterations. **(06 Marks)**

6 a. Using the Lagrange' formula, find the interpolating polynomial that approximates to the function described by the following table : **(07 Marks)**

X	0	1	2	3	4
f(x)	3	6	11	18	27

Hence find f(0.5) and f(3.1)

b. A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of t (in seconds)

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and angular acceleration of the rod at $t = 0.4$ second. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by using the Simpson's (⅜)th rule, dividing the interval into 3 equal parts. Hence find an approximate value of $\log\sqrt{2}$. **(06 Marks)**

7 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

b. Solve the variational problem :

$\delta \int_0^1 (x + y + y'^2)dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$. **(07 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is

$S = \int_{x_1}^{x_2} \sqrt{x(1+y'^2)}\, dx$. **(06 Marks)**

8 a. Find the Z-transform of

i) $3n - 4\sin\frac{n\pi}{4} - 5a^2$

ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. **(07 Marks)**

b. Obtain the inverse Z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(07 Marks)**

c. Solve the difference equation $u_{n+2} - 5u_{n+1} + 6u_n = 2$, with $u_0 = 3$, $u_1 = 7$ using z-transforms. **(06 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, December 2010

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the n^{th} derivative of $\log(ax + b)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(1+3x+2x^2)}$. **(07 Marks)**

c. If $x = \sin t$ and $y = \text{cons}\ mt$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y_n = 0$. **(07 Marks)**

2 a. Show that the following pair of curves intersect each other orthogonally.
$r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$. **(06 Marks)**

b. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

c. Find the first five terms of the Maclaurin series of $f(x) = \log \sec x$. **(07 Marks)**

3 a. If $u = e^{ax - by}\sin(ax + by)$, show that $b\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 2abu$. **(06 Marks)**

b. If $u = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find $\frac{du}{dx}$ when $x = y = a$. **(07 Marks)**

c. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that,
$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$ **(07 Marks)**

4 a. Obtain the reduction formula for $\int \cos^n x\, dx$, where n is a positive integer. **(06 Marks)**

b. Show that $\int_0^{\pi} \frac{\sqrt{1-\cos\theta}}{1+\cos\theta}\sin^2\theta\, d\theta = \frac{8\sqrt{2}}{3}$. **(07 Marks)**

c. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y\, dy\, dx$. **(07 Marks)**

5 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(07 Marks)**

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

6 a. Solve $(e^4 + 1)\cos x\, dx + e^4 \sin x\, dy = 0$. **(06 Marks)**

b. Solve $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)ds + x\sec^2\left(\frac{y}{x}\right)dy = 0$. **(07 Marks)**

c. Solve $(x + \tan y)\, dy = \sin 2y\, dx$. **(07 Marks)**

7 a. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-2x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=\cos 3x$. **(07 Marks)**

c. Solve $(D^2 - 5D + 1)y = 1 + x^2$. **(07 Marks)**

8 a. Prove that $(1 + \cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. **(06 Marks)**

b. Use Demoivre's theorem and solve the equation $x^4 - x^3 + x^2 + 1 = 0$. **(07 Marks)**

c. Expand $\cos^8\theta$ in a series of cosine of multiples of θ. **(07 Marks)**

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